Nuclear Measurement Techniques

PHYS-H-407

Teacher: N. Pauly

Course organization

- Theory:
  - 2 ECTS
  - 4 questions on theory during written examination → 60% of the final note
  - Slides available on http://metronu.ulb.ac.be/pauly_cours.html
- Laboratories:
  - 4 ECTS
  - Organization: M. Ciccarelli (Maureen.Ciccarelli@ulb.be)
  - 25% of final note → Laboratory reports
  - 1 question during written examination → 15% of final note

References:

- S. Tavernier, *Experimental techniques in nuclear and particle physics*, Springer-Verlag, 2010. Full text: http://www.springer.com/physics/particle+and+nuclear+physics/book/978-3-642-00828-3
- P. Sigmund, *Particle penetration and radiation effects*, Springer-Verlag, 2006. Full text: http://www.springer.com/physics/particle+and+nuclear+physics/book/978-3-540-31713-5
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- G.F. Knoll, *Radiation detection and measurement (4 ed.),* Wiley, 2010.
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Contents

Part I:
Reminders

Reminders

- Relativity
- Statistics
- Radioactive filiation

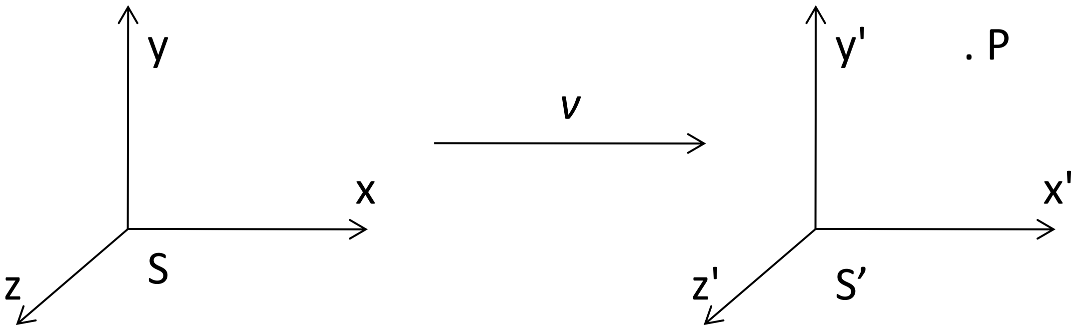
Fundamental postulates of relativity

1. *Principle of relativity* or *principle of Galilean invariance* (Poincarré, 1905): The laws of physics are identical, i.e. have identical mathematical expression, in all inertial frames of reference or Galilean reference frames (frames of reference that describe time and space homogeneously, isotropically, and in a time-independent manner; all inertial frames are in a state of constant, rectilinear motion with respect to one another)
2. *Universality of light velocity* (Einstein, 1905): The light velocity in vacuum is constant, isotropic and has same measurement in all inertial frames of reference in relative motion. This velocity does not depend on the motion of the source (light velocity: c = 299 792 458 $ms^{-1} \approx 3\ 10^{8}\ ms^{-1}$)

→ The mass m_0 and the charge q of a particle are proper characteristics of the particle and are thus invariant for a change of inertial frame of reference

Transformation of Galileo

Consider point P in 2 inertial frames: S and S' (P is at rest in S')



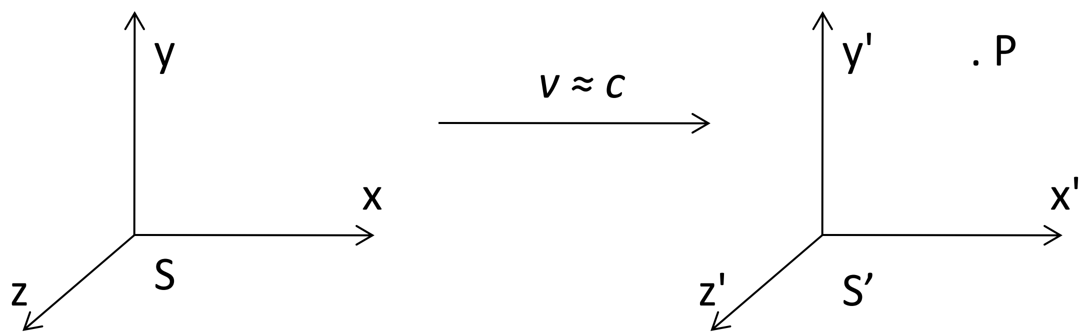
The frame S' moves with velocity *v* relative to the frame S along x axis. According to the Galilean transformation (non relativistic), we have for any point P:

$$\begin{cases} x' = x - vt \\ y' = y \\ z' = z \\ t' = t \end{cases}$$

For a particle of mass m and velocity v: $T=m_0v^2$ $p=m_0v$

Not valid for electromagnetism

Transformation of Lorentz (1)



For high velocities (near *c*) → Lorentz transformation (the light velocity *c* is the same in both frames)

$$\begin{cases} x' = \dfrac{x - vt}{\sqrt{1-(v/c)^2}} \\ y' = y \\ z' = z \\ t' = \dfrac{t - vx/c^2}{\sqrt{1-(v/c)^2}} \end{cases}$$

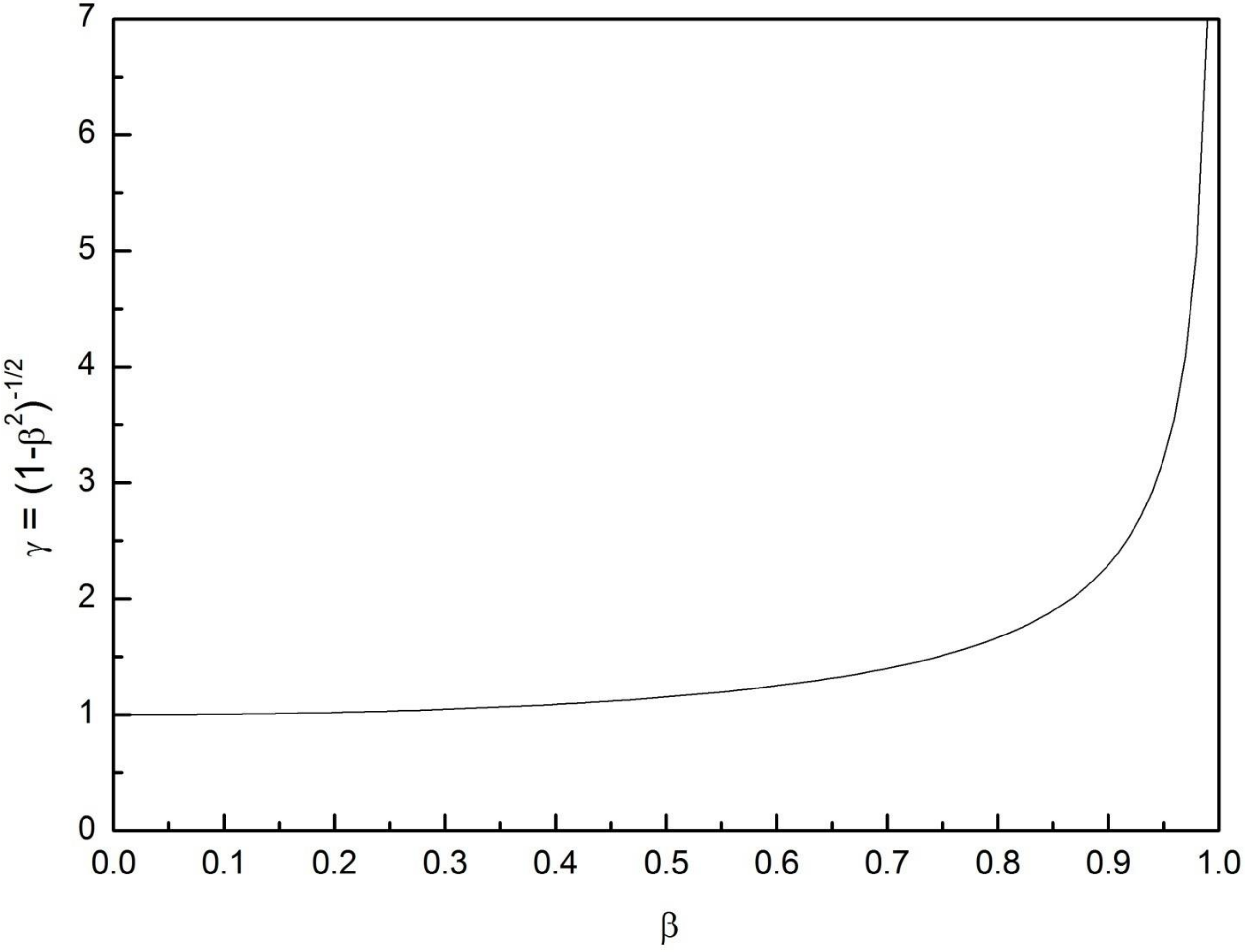
Transformation of Lorentz (2)

$$\begin{cases} x' = \gamma(x - \beta ct) \\ y' = y \\ z' = z \\ ct' = \gamma(ct - \beta x) \end{cases}$$

with

$$\begin{cases} \beta = \dfrac{v}{c} \\ \gamma = \dfrac{1}{\sqrt{1-(v/c)^2}} \end{cases}$$

Expression of γ as a function of β



Interval between 2 events

- In a 4D-space (3 spatial coordinates + time) → one event = one point → point of universe
- One moving particle describing a line in this 4D-space → line of universe
- The interval s_{12} between 2 events 1 and 2 is described by

$$s_{12}^2 = c^2(t_2 - t_1)^2 - (x_2 - x_1)^2 - (y_2 - y_1)^2 - (z_2 - z_1)^2$$

- s_{12} is the same in all inertial systems of reference → invariant
- For 2 close events → interval ds →

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 = \text{invariant}$$

Four-vector space-time

- A four-vector space-time $\overline{S}$ is defined as all real components *ct, x, y, z* → the square of its modulus is defined by →

$$S^2 = (ct)^2 - x^2 - y^2 - z^2$$

- S^2 is invariant for all Lorentzian transformations
- We consider 2 inertial frames S and S' (with the event E_1 as the origin of space and time in both frames) → if we consider a second event E_2 with coordinates *x,y,z,t* in S and *x',y',z',t'* in S' (four-vector space-time) → the quadratic form

$$s^2 = c^2t^2 - x^2 - y^2 - z^2 = c^2t'^2 - x'^2 - y'^2 - z'^2$$

is an invariant

Four-vector (in general)

- In a general way→ a four-vector $\overline{X}$ is all real components x_0, x_1, x_2, x_3 which follow the same transformation than the space-time coordinates for a change of inertial frame of references
- They have as properties:
  - The square of the amplitude $X^2 = x_0^2 - x_1^2 - x_2^2 - x_3^2$ of a four-vector is invariant
  - The linear combination $a\overline{X} + b\overline{Y}$ of 2 four-vectors is a four-vector
  - Le scalar product $\overline{X}.\overline{Y} = x_0 y_0 - x_1 y_1 - x_2 y_2 - x_3 y_3$ of 2 four-vector is invariant

Time dilatation

Consider 1 material point with velocity *v* (along x) measured in frame S → in time interval dt: it is moving to dx →

$$ds^2 = c^2dt^2 - dx^2 = c^2dt^2(1-\beta^2) = \text{invariant}$$

For observer in frame S' → point is fixed →

$$ds^2 = c^2dt'^2 = \text{invariant}$$

⇒ $$dt' = dt\sqrt{1-\beta^2} = \frac{dt}{\gamma}$$

⇒ $$\Delta t = \gamma \Delta t'$$

$\Delta t' = \Delta\tau_0$ is the *proper time* of the material point

⇒ For observer at rest (in S), the time interval Δt is always larger than the proper time

Example of time dilatation

- At Fermilab, pions π^+ are created with kinetic energy T=200 GeV → they are moving on 300m with a loss < 3%.
- The loss is due to the disintegration of π^+ → proper lifetime τ_0 = 26.0 ns (at rest)
- If its lifetime would be the same for the lab observer (in S), the fraction of surviving pions after d = 300m at a velocity v ≈ 0.99c would be

$$\exp\left(-d/c\tau_0\right) = 10^{-17} \ll 0.97$$

- But at 200 GeV → γ = 1433 → $t_{lab} = \gamma\tau_0 \approx 37\ \mu s$ →

$$\exp\left(-d/c\tau_{lab}\right) \simeq 0.97$$

cqfd

Length contraction

- An object has a fixed length l_0 (proper length) for an observer in S'
- An observer at rest sees the object moving with a velocity v (parallel to the object)
- The length l is always smaller than l_0:

$$l = \frac{l_0}{\gamma}$$

→ The length of the object in motion is always smaller than its proper length

Example of length contraction

- Cosmic muons μ are produced in the outer part of atmosphere by cosmic radiations (mainly protons) with velocity $v \approx c$

- The mean life-time of muon is τ_μ = 22 μs → they have to disintegrate after travelling a mean distance $d = c\tau_\mu$ = 660 m → no one could reach Earth surface

- In reality → contraction of the width of terrestrial atmosphere → with $\gamma \approx 1000$ → the width of the atmosphere of ≈ 10 km is « seen » by the μ as a width of ≈ 10 m

Relativistic kinematic

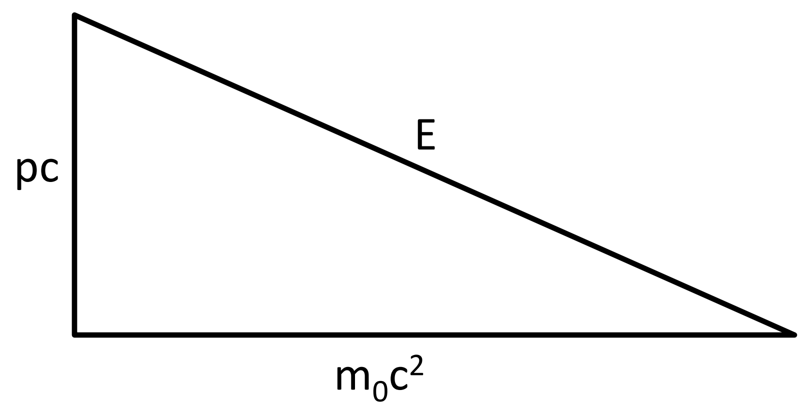
- A particle with mass at rest m_0, in motion with a velocity $\overrightarrow{v}$ in an inertial frame at rest is characterized by:
  - a momentum $\overrightarrow{p} = \gamma m_0 \overrightarrow{v}$
  - a total energy $E = \gamma m_0 c^2$
  - a kinetic energy: $T = E - m_0c^2 = (\gamma - 1)m_0c^2$

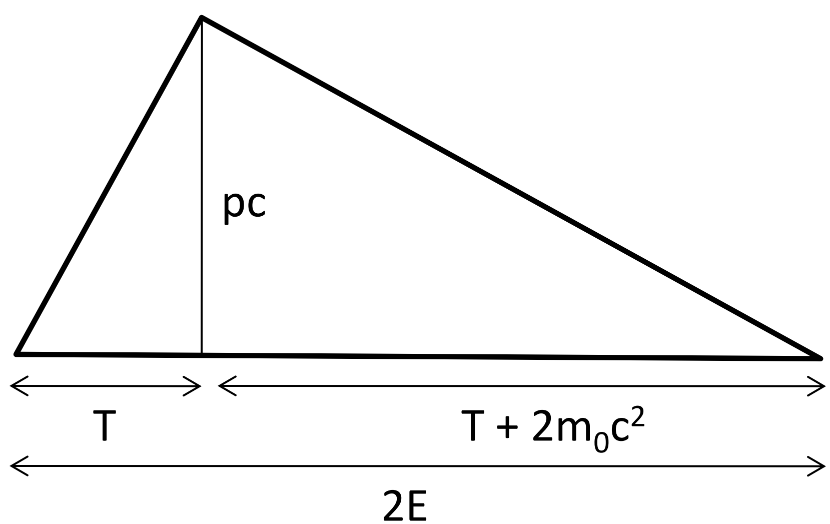
- We have the following relations between E, T and p:

$$\overrightarrow{p} = \frac{E\overrightarrow{v}}{c^2} \qquad E = \sqrt{p^2c^2 + m_0^2c^4} \qquad pc = \sqrt{T(T + 2m_0c^2)}$$

- For a photon → $v = c$ → $E = pc$

Geometric illustration of kinematic relations





Four-vector energy-momentum

- The 4 components (E/c, p_x, p_y, p_z) also constitute a four-vector: energy-momentum four-vector $\overline{P}$
- The square of its modulus is invariant for a change of inertial frame:

$$P^2 = \frac{E^2}{c^2} - p^2 = \text{constant}$$

- In a frame for which the particle is at rest →

$$P^2 = m_0^2 c^2$$

- We deduce →

$$E^2 = p^2 c^2 + m_0^2 c^4$$

Four-vector energy-momentum for a set of particles

- We consider a set of free particles without interaction → each particle is characterized by a four-vector space-time and by a four-vector energy-momentum
- The resulting $\overline{P} = \sum_k \overline{P}_k$ is also a four-vector characterized by the algebraic sum of the four-vector components of all particles

$$\overrightarrow{p} = \sum \overrightarrow{p_k} \quad \text{and} \quad E = \sum E_k$$

- Property of four-vector → P^2 invariance →

$$P^2 = \frac{E^2}{c^2} - p^2 = \text{invariant}$$

- Useful relation of collision or disintegration studies

Example: Particle disintegration (1)

- Let disintegration be A → B + C with A initially at rest in the laboratory frame (which is also the center of mass frame) →

$$0 = \overline{p}_B + \overline{p}_C \rightarrow |p_B| = |p_C|$$

- The four-vector energy-momentum invariance implies →

$$\frac{E_A^2}{c^2} = \frac{(E_B + E_C)^2}{c^2} \Rightarrow m_A c^2 = E_B + E_C$$

- Moreover →

$$\begin{cases} E_B^2 = & p_B^2 c^2 + m_B^2 c^4 \\ E_C^2 = & p_C^2 c^2 + m_C^2 c^4 \end{cases}$$

- By subtraction → $E_B^2 - E_C^2 = (m_B^2 - m_C^2)c^4$
- Dividing by $m_A c^2 = E_B + E_C$

Example: Particle disintegration (2)

- We thus obtain → $$E_B - E_C = \frac{m_B^2 - m_C^2}{m_A} c^2$$
- And finally →

$$\begin{cases} E_B = & \dfrac{m_A^2 + m_B^2 - m_C^2}{2m_A} c^2 \\ E_C = & \dfrac{m_A^2 + m_C^2 - m_B^2}{2m_A} c^2 \end{cases}$$

- As $p_{B,C}^2 \geq 0$ → we deduce $m_A \geq m_B + m_C$ → $m_A c^2 = m_B c^2 + m_C c^2 + T$ with T, the total kinetic energy in the center of mass frame

Example: Threshold energy (1)

- The threshold energy for the production of Q particles during an inelastic collision is the minimal kinetic energy of the N incident particles that allows to create particles at rest in the center of mass frame
- We consider as instance the minimal kinetic energy of 1 particle with mass m_1 colliding with 1 particle at rest with mass m_2, to form Q particles with mass m_j
- In the laboratory frame, S, before collision →

$$P^2 = \frac{(E_1 + m_2 c^2)^2}{c^2} - p_1^2$$

- In the center of mass frame, S', after collision →

$$\sum_j^Q p_j = 0 \rightarrow P^2 = \frac{(\sum_j^Q m_j c^2)^2}{c^2}$$

Example: Threshold energy (2)

- With

$$p_1^2 c^2 = E_1^2 - m_1^2 c^4$$

- We know

$$E_1 = T_{min} + m_1 c^2$$

- By calculation →

$$T_{min} = \frac{\left(\sum_j^Q m_j c^2\right)^2 - (m_1 c^2 + m_2 c^2)^2}{2 m_2 c^2}$$

- For the collision between 2 protons (one at rest) →

$$p + p \rightarrow p + p + p + \overline{p}$$

- With m_p = 938 MeV/c^2 → T_{min} = 6 mc^2 → T_{min} = 5.63 GeV

Remark on the relativistic limit

- If the velocity of a particle is « close » to velocity of light, relativistic calculations have to be used for the particle
- What means « close » to the light velocity? Difficult…
- It is usual easier to consider the kinetic energy of the particle *T*:

  If $T > (1/200)m_0c^2$ (m_0: mass at rest of the particle), relativistic calculations have to be used (200 is an arbitrary value depending on applications and on needed precision)

examples: for electron → $(1/200)m_ec^2$ = 2.56 keV

for proton → $(1/200)m_pc^2$ = 4690 keV

Reminders about statistic

Consider a process with a number of successes x resulting from a given number of trials n. Each trial is a binary process. We suppose the probability of success as p.

Three statistical models are important in this course:

- The Binomial distribution
- The Poisson distribution
- The Gaussian (or Normal) distribution

Binomial distribution

It is applicable to all constant-p processes.

$$P(x) = \frac{n!}{(n-x)!x!}p^x(1-p)^{n-x}$$

Some properties:

$$\sum_{x=0}^{n} P(x) = 1$$

$$\overline{x} = \sum_{x=0}^{n} xP(x) = pn$$

$$\sigma^2 = \sum_{x=0}^{n} (x-\overline{x})^2 P(x) = np(1-p)$$

Poisson distribution

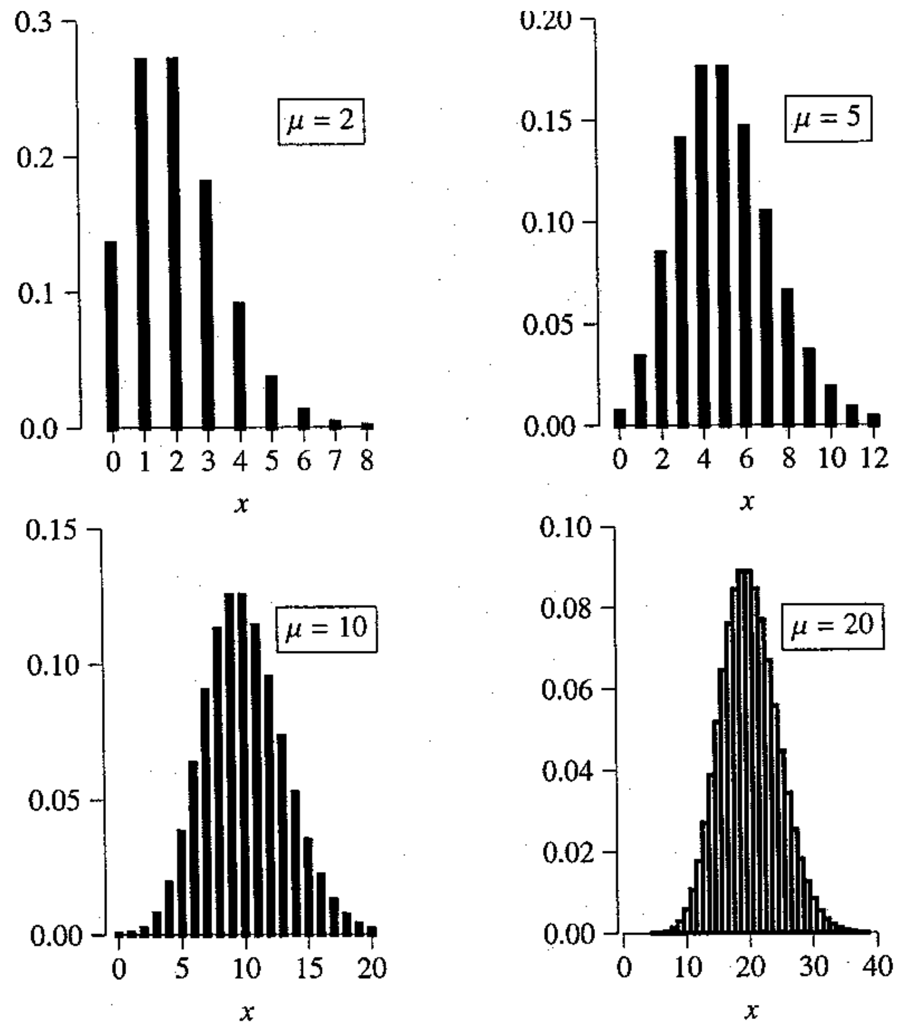
It is a simplification of the binomial distribution under the conditions that *n* is large and *p* is small

$$P(x) = \frac{n!}{(n-x)!x!}p^x(1-p)^{n-x} = \frac{n!}{(n-x)!n^x}\frac{1}{x!}(np)^x e^{-pn} = \frac{(pn)^x e^{-pn}}{x!}$$

$$P(x) = \frac{(\overline{x})^x e^{-\overline{x}}}{x!} \quad \text{with} \quad \overline{x} = pn$$

$$\sigma^2 = pn = \overline{x}.$$

Examples of Poisson distributions



Poisson distribution for radioactive processes

4 conditions: - atoms are identical

- they are independent
- their mean life is long
- their number is large

→ probability for x disintegrations in a time interval T:

$$P_x(T) = \frac{(aT)^x}{x!} e^{-aT}$$

With a is the average rate of disintegrations per time unit → $\overline{x}$ = aT

Interval distribution for radioactive processes: Erlang distribution (1)

- Probability that there is no event in a time interval [0,t] is e^{-at}
- Probability that there is exactly one event in dt is adt
- Combined probability to observe the first disintegration in the interval [t, t+dt] is

$$f_1(t) = e^{-at} a dt$$

- $f_1(t)$ is the probability density of the random variable t defined as the time between two successive disintegrations
- We have now to determine the probability density $f_k(t)$ of the time interval t between one arbitrary disintegration and the k^{th} following

Interval distribution for radioactive processes: Erlang distribution (2)

- Cumulative distribution function $F_k(t)$ (probability to observe *k* disintegrations in a time interval < *t* or equivalently probability to obtain at least k disintegrations in the time interval [0 t]):

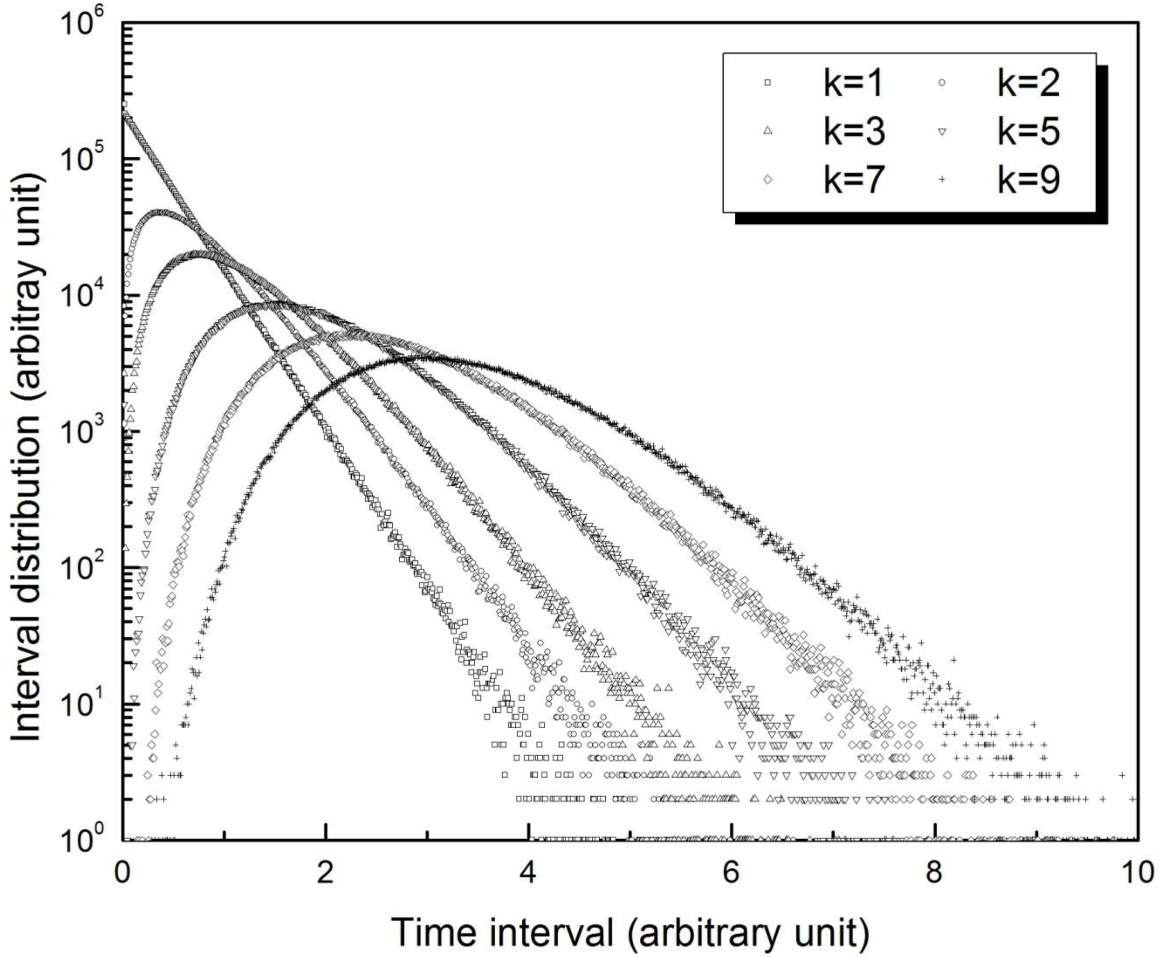
$$F_k(t) = \int_0^t f_k(t)dt = 1 - \sum_{x=0}^{k-1} \frac{(at)^x}{x!} e^{-at}$$

- Knowing that

$$\sum_{x=0}^{k-1} \frac{(at)^x}{x!} e^{-at} = \int_{at}^{\infty} \frac{z^{k-1}}{(k-1)!} e^{-z} dz$$

$$\Rightarrow \quad f_k(t)dt = \frac{(at)^{k-1}}{(k-1)!} e^{-at} a dt$$

Examples of Erlang distributions



Time intervals distributions for (□) *k = 1;* (o) *k = 2;* (Δ) *k = 3;* (∇) *k = 5;* (◊) *k = 7* and (+) *k = 9*

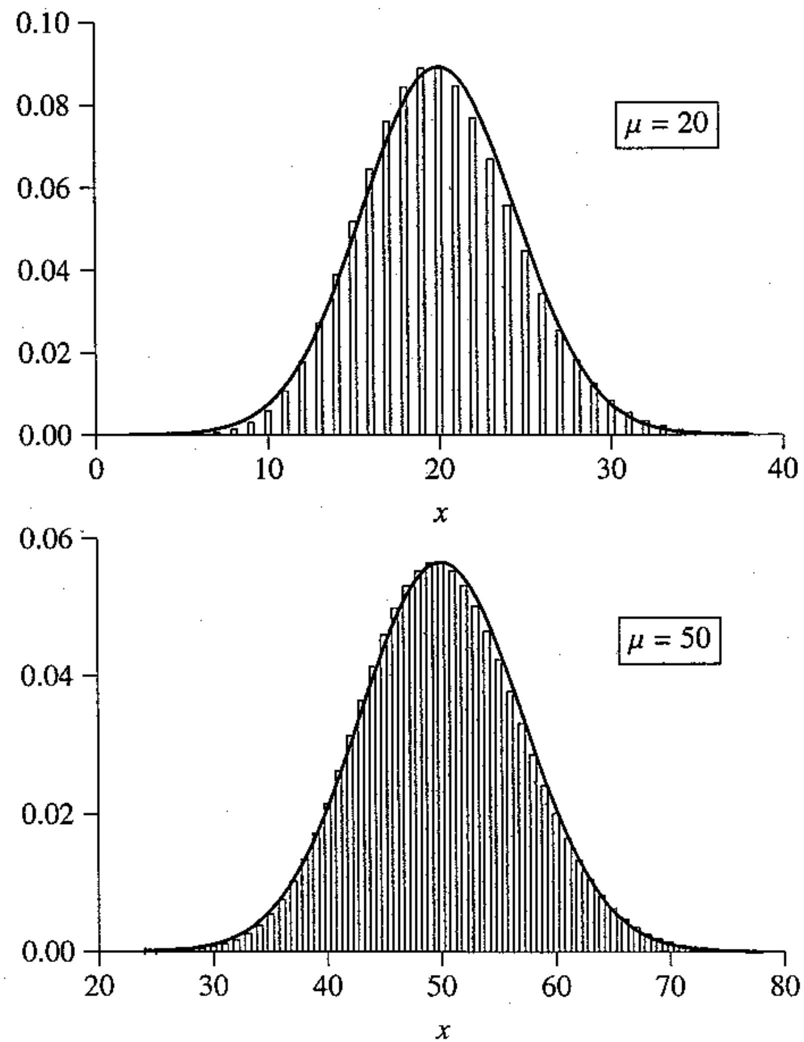
Gaussian (or Normal) distribution

If p is small and the mean value of the distribution is large (>∼20) →

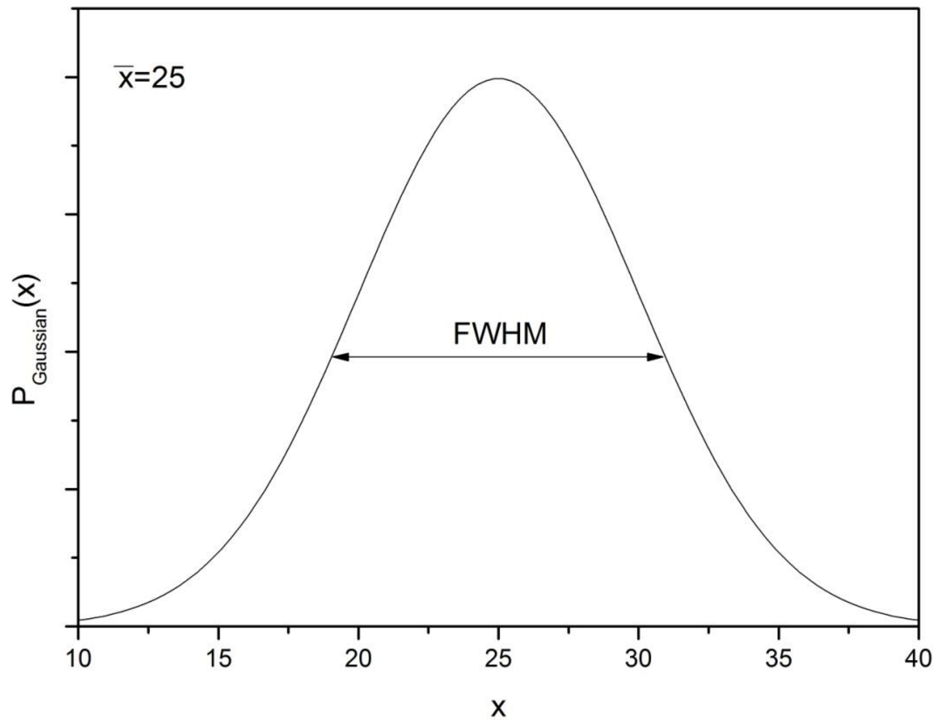
$$P(x) = \frac{1}{\sqrt{2\pi\overline{x}}} \exp\left(-\frac{(x-\overline{x})^2}{2\overline{x}}\right)$$

Valid for any situation in which we accumulate more than a few counting during the course of the measurement

Comparison Poisson - Gaussian



Gaussian (or Normal) distribution (2)



$$\text{Full width at half maximum} = \text{FWHM} = 2\sqrt{2\ln 2}\sigma = 2.355\sigma \text{ with } \sigma = \sqrt{\overline{x}}$$

Radioactive filiation: Disintegration constant

- Probability of disintegration per unit time: λ = disintegration constant
- λ*dt* is the disintegration probability of a nucleus in the time interval *dt*
- Application of Poisson distribution → survival probability of a nucleus at time t (if existing in $t = 0$)→

$$P_0(t) = e^{-\lambda t}$$

- If N_0 is the initial number (at $t = 0$) of nuclei → the number of survival nuclei $N(t)$ at time t is

$$N(t) = N_0 e^{-\lambda t}$$

Half-life and activity

- The half-life $T_{½}$ is time is time taken for half the radionuclide's atoms to decay →

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

- Activity *A(t)* at time *t* is defined as the mean number of disintegrations per time unit →

$$A(t) = \lambda N(t)$$

- The activity unit is Becquerel (Bq) → 1 Bq = 1 disintegration per second (old unit → Curie (Ci) corresponding to the activity of 1 g of ^{226}Ra → 1 Ci = 3.7 $\times$ 10^{10} Bq)

Radioactive filiation (1)

- We suppose → $X_1 \overset{\lambda_1}{\rightarrow} X_2 \overset{\lambda_2}{\rightarrow} X_3$
- The number of X_1 (« parent ») decreases following an exponential equation →

$$\frac{dN_1}{dt} = -\lambda_1 N_1 \rightarrow N_1(t) = N_1(0)e^{-\lambda_1 t}$$

- The number of X_2 (« daughter ») increases due to disintegration of X_1 and disintegrates with the disintegration constant λ_2 →

$$\frac{dN_2}{dt} = -\lambda_2 N_2 + \lambda_1 N_1 = -\lambda_2 N_2 + \lambda_1 N_1(0)e^{-\lambda_1 t}$$

- The solution is→

$$N_2(t) = N_2(0)e^{-\lambda_2 t} + \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1(0)\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

Radioactive filiation(2)

- The number of X_3 changes as

$$\frac{dN_3}{dt} = \lambda_2 N_2$$

$$\Rightarrow$$

$$N_3(t) = N_3(0) + N_2(0)\left(1 - e^{-\lambda_2 t}\right) + N_1(0)\left(1 - \frac{\lambda_2 e^{-\lambda_1 t} - \lambda_1 e^{-\lambda_2 t}}{\lambda_1 - \lambda_2}\right)$$

- Practically → measures of activities $A_1 = \lambda_1 N_1$ and $A_2 = \lambda_2 N_2$ → assuming $N_2(0) = N_3(0) = 0$ →

$$A_1(t) = A_1(0)e^{-\lambda_1 t}$$ and $$A_2(t) = \frac{\lambda_2}{\lambda_2 - \lambda_1} A_1(0)\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

Equilibria (1)

- We note that $A_1(t)$ is maximum at $t = 0$ and zero at $t = \infty$ and that $A_2(t)$ is zero at $t = 0$ and $t = \infty$ → $A_2(t)$ has a maximum for $dA_2(t)/dt = 0$ →

$$\frac{d(A_2)}{dt} = 0 = -\lambda_1 e^{-\lambda_1 t_m} + \lambda_2 e^{\lambda_2 t_m}$$

$$\Rightarrow \quad t_m = \frac{\ln \lambda_2/\lambda_1}{\lambda_2 - \lambda_1}$$

- This maximum happens when the activities of parent and daughter are equal → $A_1(t_m) = A_2(t_m)$

$$e^{-\lambda_1 t_m} = \frac{\lambda_2}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t_m} - e^{\lambda_2 t_m}\right)$$

$$\Rightarrow \quad t_m = \frac{\ln \lambda_2/\lambda_1}{\lambda_2 - \lambda_1}$$

Equilibria (2)

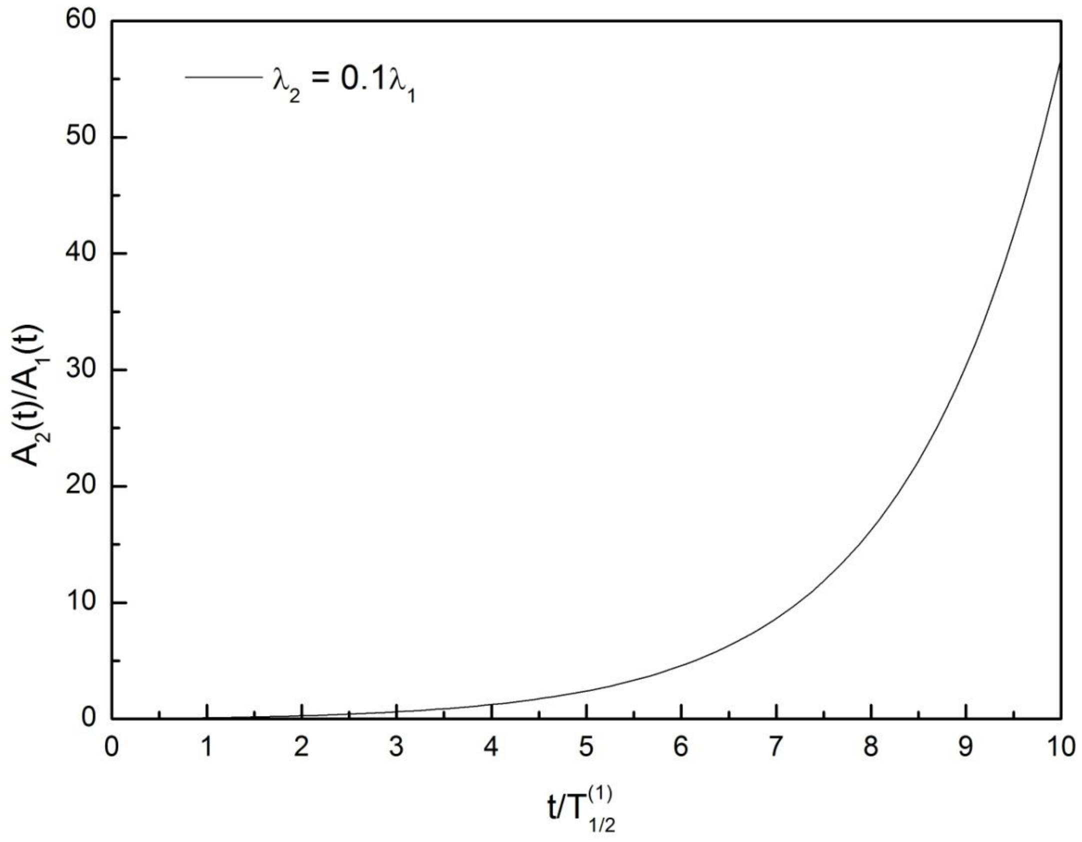
- At t_m → we have « *ideal equilibrium*»
- The ratio of activities of X_2 and X_1 is →

$$\frac{A_2(t)}{A_1(t)} = \frac{\lambda_2}{\lambda_2 - \lambda_1}\left(1 - e^{-(\lambda_2-\lambda_1)t}\right)$$

- For $t < t_m$ → always $A_1 > A_2$
- For $t > t_m$ → always $A_1 < A_2$
- The specific relation between parent and daughter depends on the relative values of their disintegration constants→ 3 cases →
  1. $\lambda_2 < \lambda_1$
  2. $\lambda_2 > \lambda_1$
  3. $\lambda_2 \gg \lambda_1$

Non-equilibrium: $\lambda_2 < \lambda_1$

- X_1 isotopes disintegrate faster than filiation products $X_2 \rightarrow$ the ratio of activities increases without limit

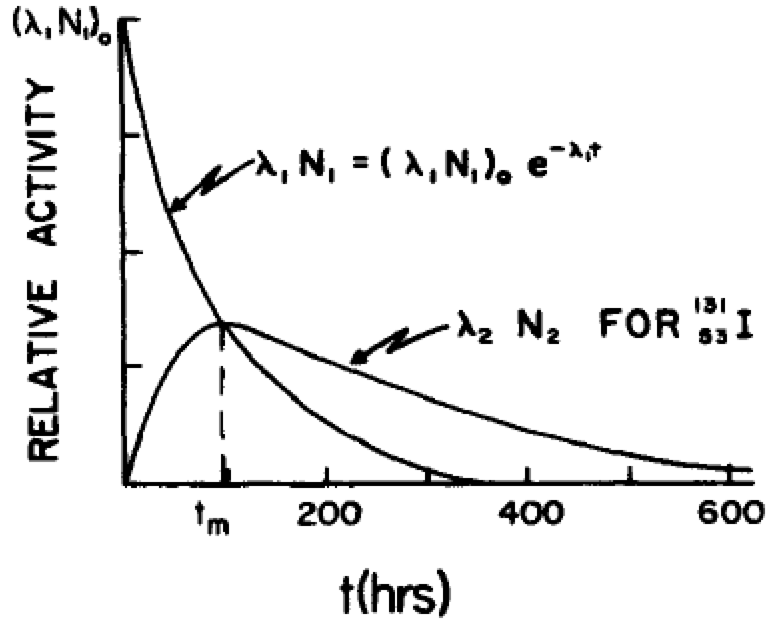


Example with $\lambda_2 < \lambda_1$

- Disintegration of metastable tellurium →

$$ {}^{131m}_{52}\mathrm{Te} \xrightarrow{T_{1/2}\,=\,30h} {}^{131}_{53}\mathrm{I} \xrightarrow{T_{1/2}\,=\,193h} {}^{131}_{54}\mathrm{Xe} $$

- We have thus → $\lambda_1 = 2.31\ 10^{-2}\ h^{-1}$ and $\lambda_2 = 3.59\ 10^{-3}\ h^{-1}$

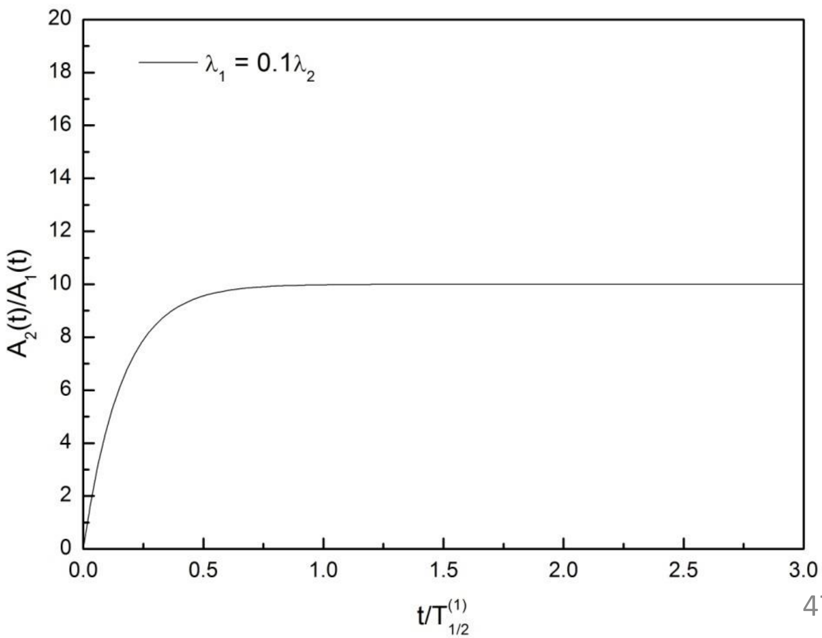


Transient equilibrium: $\lambda_2 > \lambda_1$

- The activities ratio increases as a function of time and reaches a constant value → for $t \to \infty$:

$$\frac{A_2(t)}{A_1(t)} \simeq \frac{\lambda_2}{\lambda_2 - \lambda_1}$$

- The daughter activity decreases at the same rate as that of the parent → this equilibrium is called transient equilibrium



Secular equilibrium: $\lambda_2 \gg \lambda_1$

- The activities ratio increases as a function of the time and reaches 1 pour $t \rightarrow \infty$:

$$\frac{A_2(t)}{A_1(t)} \simeq 1$$

- The parent and daughter activities become equal → secular equilibrium
- Example → disintegration of radium →

$$^{226}_{88}\mathrm{Ra} \overset{T_{1/2} = 1602 \text{ an}}{\longrightarrow} {}^{222}_{86}\mathrm{Rn} \overset{T_{1/2} = 3.8 \text{ j}}{\longrightarrow} {}^{218}_{84}\mathrm{Po}$$

- We have → λ_1 = 1.18 10^{-6} j^{-1} and λ_2 = 1.81 10^{-1} j^{-1}